The Factor Theorem

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$x - r$ is a factor of a polynomial $P(x)$ if and only if r is a root of $P(x)$.

$(x - r)$ is a factor of $P(x)$ if when using long division, the remainder is zero.
r is a x-intercept of $P(x)$ if, when using synthetic division, the remainder is zero.

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

2) Given the following equations, determine the solutions for each.

a) $0 = (x-2)(x+5)(x+1)$

$x-2=0$ $\quad x+5=0$ $\quad x+1=0$

$x=2$ $\quad x=-5$ $\quad x=-1$

b) $(x+2)(x^2-25)=0$

$x+2=0$ $\quad x^2-25=0$

$x=-2$ $\quad$ $+25 \quad +25$

$\sqrt{x^2} = \sqrt{25}$

$|x| = 5$

$x = \pm 5$

c) $x(x^2+2x+2)(x-5)=0$

$x=0$ $\quad -1 \pm i$ $\quad x=5$

d) $(x-9)(x-2)(x-6)(x+4)=0$

$x=9$ $\quad x=2$ $\quad x=6$ $\quad x=-4$

3) Given the following solutions, determine the factored form of the equation having the given solutions.

a) $x=2,\ x=-7,\ x=3$

$y=(x-2)(x+7)(x-3)$

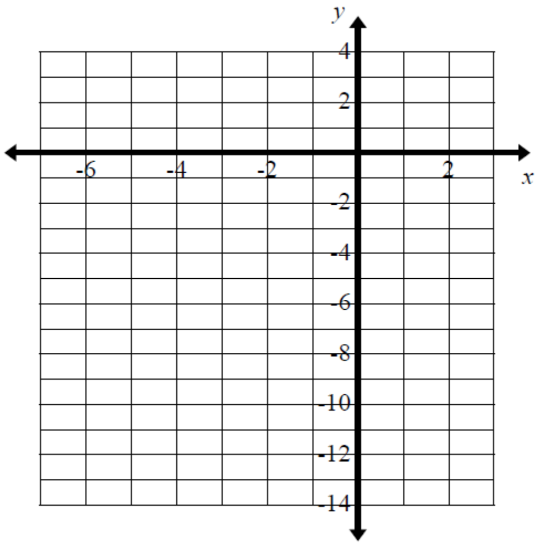
b) $x=-1,\ x=0,\ x=2,\ x=3$

$y=(x+1)(x)(x-2)(x-3)$

$y=x(x+1)(x-2)(x-3)$

4) Use a graphing calculator to graph the function $y = x^3 + 6x^2 + 5x - 12$. Use the graph of the function to find ALL the solution(s) for $x^3 + 6x^2 + 5x - 12 = 0$. All answers must be ~~exact answers~~.

3 solutions



$x=-4$

$x=-3$

$x=1$

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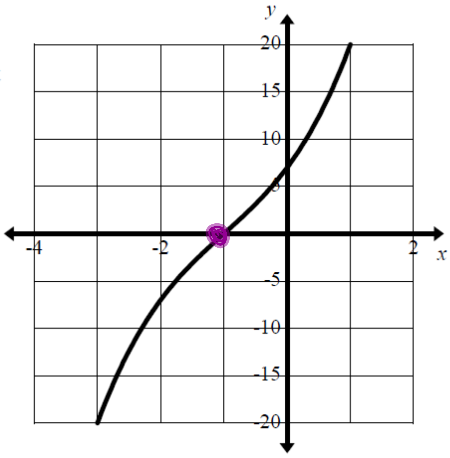
5) Find the zero(s) for $f(x) = x^3 + 3x^2 + 9x + 7$.

Note: this graph has only one x-intercept at $x = -1$.

However, this is a **cubic polynomial** and therefore has **three zeros**. Find the other two zeros. All answers must be exact, no approximate answers will be accepted.



a) Identify the factor that created the zero shown on the graph: $(x+1)$

Just like dividing 12 by 4 to find an additional factor of 3, you can divide a polynomial by a factor to find additional factors.

b) Use either long division or synthetic division to divide the polynomial by the information gathered from the graph regarding zeros of the function.

i) To use long division, what would you divide by? $x+1$

ii) To use synthetic division, what value would you place in the top left corner? -1

iii) Complete the division of the polynomial using either long division or synthetic division. Record your thinking below and record the quotient here, setting it equal to zero. x^2+2x+7

iv) What degree is the quotient? 2 You have several methods to solve an equation of this nature. List them here:

- graphing
- factoring
- quadratic formula
- square roots

no x-ints →

v) Using one of the above methods, find the remaining two zeros.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a = 1$
$b = 2$
$c = 7$

$$x = \frac{-(2) \pm \sqrt{(2)^2-4(1)(7)}}{2(1)} = \frac{-2 \pm \sqrt{-24}}{2}$$

$i\sqrt{4}\sqrt{6}$

$2i\sqrt{6}$

$$x = \frac{-2 \pm 2i\sqrt{6}}{2}$$

$$x = -1 \pm i\sqrt{6}$$

Solutions $= -1, -1+i\sqrt{6}, -1-i\sqrt{6}$

6) Find the zero(s) for $f(x) = x^4 + 7x^3 + 8x^2 - 13x - 3$.

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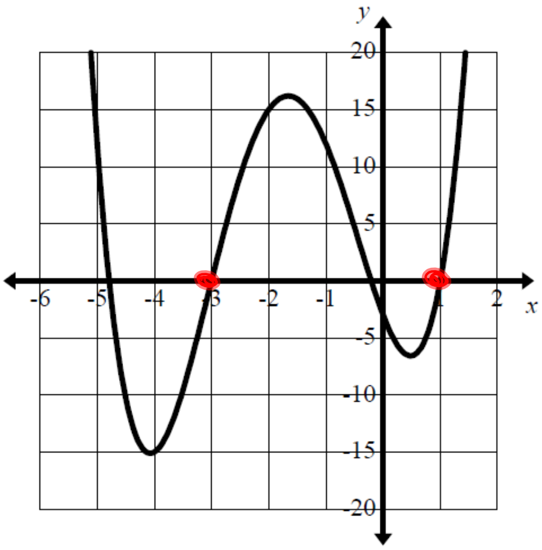
Note: this graph has only four x-intercepts at $x = 1$ and $x = -3$ can be easily found from the graph. The other two intercepts are irrational numbers. Find them on your graphing calculator and see for yourself.



If a student were absent from class today, write a description of a method to find exact values of the other two intercepts.

Find the other two zeros. All answers must be exact. No approximate answers will be accepted.

$$\begin{array}{r|rrrrr} 1 & 1 & 7 & 8 & -13 & -3 \\ & \downarrow & 1 & 1 & 16 & 3 \\ \hline & 1 & 8 & 16 & 3 & 0 \end{array}$$

$f(x) = 1x^3 + 8x^2 + 16x + 3$

$$\begin{array}{r|rrrr} -3 & 1 & 8 & 16 & 3 \\ & \downarrow & -3 & -15 & -3 \\ \hline & 1 & 5 & 1 & 0 \end{array}$$

$f(x) = 1x^2 + 5x + 1$

$a = 1$
$b = 5$
$c = 1$

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{21}}{2}$$

$$\boxed{\text{Solutions } x = 1, -3, \frac{-5 \pm \sqrt{21}}{2}}$$

Note that the two irrational solutions came as a pair. This is due to how the quadratic formula works. The same is true for imaginary solutions. **Irrational and imaginary solutions will always come in pairs.** However, imaginary solutions will not able to be seen on the graph of the function. Remember, due to the Fundamental Theorem of Algebra, the total number zeros (rational, irrational, and imaginary) must equal to the degree of the polynomial.

HOMEWORK:

	6.2 D		6.2 D #1-9 (P-105)		☹	😐	☺
	6.2 E		6.2 E #1-7, 12, 13 (P-109)		☹	😐	☺
	6.3 A & 6.3 B	I can solve polynomial equations.	6.3A ~~#1-3, 6~~ 1-2 (P-113)		☹	😐	☺
			6.3B ~~#2-8~~ 2,4,6 (P-118)		☹	😐	☺
5/12	6.3 C		6.3C ~~#1-6~~ #2,4,5 (P-123)		☹	😐	☺
	Review	Unit 6 Part 1 Review					

#2 part b.
needs long division
$3x+1$ is divisor